Hyperplane arrangements 2023

- Date : 11 December 2023 – 15 December 2023
- Place : Tachikawa Hall 3F, Rikkyo University, Ikebukuro Campus

11 December (Monday)

9:00 – 10:00	Opening, registration and discussion
10:00 – 11:00	Shuhei Tsujie (Hokkaido University of Education) Characteristic quasi-polynomials of root systems associated with unitary reflection groups
11:20 – 11:50	Lorenzo Giordani (Ruhr-Universität Bochum) Cohomology rings of toric wonderful models
11:50 – 13:40	Lunch Break
13:40 – 14:40	Michael DiPasquale (New Mexico State University) Saturation of the Jacobian ideal of a hyperplane arrangement in minimal degree
14:55 – 15:55	Giovanni Paolini (Bologna University) Dual Coxeter groups of rank three
16:15 – 16:45	Sakumi Sugawara (Hokkaido University) Description of Kirby diagrams for plane curve complements via braid monodromy

12 December (Tuesday)

9:30 – 10:30	Paul Mücksch (Leibniz Universität Hannover) Milnor fibrations for oriented matroids
10:50 – 11:50	Botong Wang (University of Wisconsin-Madison) Cohomology of certain $\mathbb{Z}$-local systems on hyperplane arrangement complements
11:50 – 13:40	Lunch Break
13:40 – 14:40	Toshitake Kohno (Meiji University) Homotopy 2-groupoids of hyperplane arrangements
14:55 – 15:55	Kazuki Hiroe (Chiba University) Long-Moody construction of representations of braid groups and Katz middle convolution
16:15 – 16:45	Leonie Mühlherr (Universität Bielefeld) Recent developments in graphic hyperplane arrangement theory
16:55 – 17:25	Shota Maehara (Kyushu University) Explicit description of a basis for derivation modules of some multiarrangements of type B_2

13 December (Wednesday)

9:30 – 10:30	Alexandru Suciu (Northeastern University) Hyperplane arrangements with trivial algebraic monodromy
10:50 – 11:50	Max Wakefield (The United States Naval Academy) Chain Tutte polynomials
11:50 – 18:00	Free Discussion

14 December (Thursday)

9:30 – 10:30	Tan Nhat Tran (Leibniz Universität Hannover) Vines and MAT-labeled graphs
10:50 – 11:50	Roberto Pagaria (Bologna University) Cohomology rings of abelian arrangements
11:50 – 13:40	Lunch Break
13:40 – 14:40	Graham Denham (Western University) Kirchhoff polynomials and configuration hypersurfaces
14:55 – 15:55	Nir Gadish (The University of Michigan) Letter-braiding invariants of words in groups
16:15 – 16:45	Zixuan Wang (Hokkaido University) Integral expressions for multiarrangements
16:55 – 17:25	Junyan Chu (Kyushu University) Minimal Free Resolution of Close-to-free Arrangements

15 December (Friday)

9:30 – 10:30	Gerhard Röhrle (Ruhr-Universität Bochum) Invariants and semi-invariants in the cohomology of the complement of a reflection arrangement
10:50 – 11:50	Emanuele Delucchi (Scuola Universitaria Professionale Della Svizzera Italiana) A new arrangement from metric spaces
11:50 – 13:40	Lunch Break
13:40 – 14:40	Christin Bibby (Louisiana State University) Supersolvable posets and fiber-type abelian arrangements
14:55 – 15:25	Sven Wiesner (Ruhr-Universität Bochum) Free multiderivations of connected subgraph arrangements
15:35 – 16:05	Georges Neaime (Universität Bielefeld) Reflection Groups and Artin Groups: Classical and Elliptic Theories
16:25 – 17:25	Lukas Kühne (Universität Bielefeld) Computing the moduli space of a matroid and applications to arrangements

Organizers:

Takuro Abe	(Chair, Rikkyo University)
Norihiro Nakashima	(Nagoya Institute of Technology)
Shuhei Tsujie	(Hokkaido University of Education)
Masahiko Yoshinaga	(Osaka University)

Abstracts.

Shuhei Tsujie (Hokkaido University of Education)
Title: Characteristic quasi-polynomials of root systems associated with unitary reflection groups
Abstract: Kamiya, Takemura, and Terao initiated the theory of the characteristic quasi-polynomial of an integral arrangement, which is a function counting the elements in the complement of the arrangement modulo positive integers. The characteristic quasi-polynomials of crystallographic root systems exhibit many interesting properties. In this talk, we will explore the potential generalization of these properties to non-crystallographic cases. This is joint work with Masamichi Kuroda.

Lorenzo Giordani (Ruhr-Universität Bochum)
Title: Cohomology rings of toric wonderful models
Abstract: One of the leading motifs in the theory of arrangements is to understand the interplay of algebraic, geometric and topological properties with combinatorial ones, meant as properties of the intersection structure of the arrangements.

An approach in this direction, initiated by De Concini and Procesi in the nineties, is the introduction of "wonderful" compactifications for complement spaces of subspace arrangements. The study of wonderful models has ensured the combinatorial nature of relevant topological invariants of complement spaces, namely: cohomology, rational homotopy type, and mixed Hodge structure.

Projective wonderful models were extended to the case of toric arrangements of arbitrary codimension. Their cohomology has then been studied by De Concini and Gaiffi in the "well-connected" case, in which properties holding naturally in the linear case are imposed.

In this work in progress with Roberto Pagaria and Viola Siconolfi we remove the hypothesis of well-connectedness, and offer a different presentation for the cohomology ring of the model. Our approach is closer to the combinatorial picture: we adapt the notions of blowups for semilattices and nested set complex introduced by Feichtner and Kozlov to the poset of layers of toric arrangements.

Michael DiPasquale (New Mexico State University)
Title: Saturation of the Jacobian ideal of a hyperplane arrangement in minimal degree
Abstract: Let $\mathcal{A} \subset V$ be a central hyperplane arrangement with n hyperplanes in a vector space V over a field of characteristic 0, and $J_{\mathcal{A}}$ be its Jacobian ideal. It is known that the saturation of $J_{\mathcal{A}}$, which we denote as $\mathrm{sat}(J_{\mathcal{A}})$, consists of

polynomials of degree at least $n-1$. We provide a description of the vector space $\mathrm{sat}(J_{\mathcal{A}})_{n-1}$. Our description recovers (in characteristic zero) results of Yuzvinsky on formal and k-generated arrangements. For line arrangements, we may further identify the space $\mathrm{sat}(J_{\mathcal{A}})_{n-1}$ with the infinitesimal motions of $\mathcal{A}$ regarded as a framework in the projective plane. Also special to line arrangements is a duality which connects the space $\mathrm{sat}(J_{\mathcal{A}})_{n-1}$ to the second syzygies of $J_{\mathcal{A}}$ of largest possible degree $(2n-2)$. Using these two tools we observe that a well-known example of Ziegler (in which the graded betti numbers of $J_{\mathcal{A}}$ are shown to be sensitive to geometry) fits into a broader class of generically rigid intersection lattices with infinitesimally flexible realizations. We ask a number of related open questions. This is joint work with Jessica Sidman (Amherst College) and Will Traves (Naval Academy).

Giovanni Paolini (Bologna University)
Title: Dual Coxeter groups of rank three
Abstract: In this presentation, I will discuss the combinatorics of the noncrossing partition posets associated with Coxeter groups of rank three. In particular, I will describe the techniques used to prove the lattice property and lexicographic shellability. These properties can then be used to solve several problems on the corresponding Artin groups, such as the $K(\pi, 1)$ conjecture, the word problem, the center problem, and the isomorphism between standard and dual Artin groups. This research was conducted in collaboration with Emanuele Delucchi and Mario Salvetti.

Sakumi Sugawara (Hokkaido University)
Title: Description of Kirby diagrams for plane curve complements via braid monodromy
Abstract: The notion of braid monodromy is a very useful tool to study the topology of the complement of plane algebraic curves. For example, braid monodromy tells us the information of fundamental groups and homotopy types of the complement. The Kirby diagram is a framed link diagram that describes the handle decomposition of a 4-manifold. In this talk, the speaker will show that the Kirby diagram of the plane curve complement is described by using the braid monodromy.

Paul Mücksch (Leibniz Universität Hannover)
Title: Milnor fibrations for oriented matroids
Abstract: My talk will be a report on recent joint work with Masahiko Yoshinaga. We present a new approach towards the study of Milnor fibers of hyperplane arrangements via combinatorial models of fibrations. One of our central results is a new regular CW-complex associated to any oriented matroid, sim-

ilar to Salvetti's famous complex, which, in the realizable case, is homotopy equivalent to the geometric Milnor fiber of the complexified real arrangement.

Botong Wang (University of Wisconsin-Madison)
Title: Cohomology of certain $\mathbb{Z}$-local systems on hyperplane arrangement complements
Abstract: In his Annals paper, Libgober showed that the Alexander polynomial of a hypersurface complement is controlled by the Alexander polynomial of the tubular neighborhood at infinity. Following this idea, Cohen-Dimca-Orlik proved vanishing theorems for cohomology groups of certain $\mathbb{C}$-local systems on hyperplane arrangement complements. We aim to study the $\mathbb{Z}$-local systems on hyperplane arrangement complements satisfying the vanishing conditions of Cohen-Dimca-Orlik. We discuss the possible torsions in the cohomology groups, and prove a conjecture of Sugawara. This is joint work with Yongqiang Liu and Laurentiu Maxim.

Toshitake Kohno (Meiji University)
Title: Homotopy 2-groupoids of hyperplane arrangements
Abstract: The purpose of this talk is to describe an application of higher holonomy functors to the study of homotopy 2-groupoids of the complement of hyperplane arrangements. The theory of 2-connections on principal 2-bundles and their two dimensional holonomy has been developed by Baez, Schreiber and others. This construction is based on the notion of crossed modules, which was developed by J. H. C. Whitehead for a description of 2-homotopy relative to the 1-skeleton. We obtain a categorical representation of the path 2-groupoid to the 2-Lie group. Under a 2-flatness condition we have representations of homotopy 2-groupoids. We use the theory of formal connections in the framework of Chen's iterated integrals and describe a universal expression of two dimensional holonomy. We apply this method to the cobar constructions of Orlik-Solomon algebras for hyperplane arrangements.

Kazuki Hiroe (Chiba University)
Title: Long-Moody construction of representations of braid groups and Katz middle convolution
Abstract: The Burau and Gassner representations will be one of the most famous representations of braid groups which appear in various ares of mathematics and also even in Physics. As a generalization of the homological description of Burau-Gasnner representations, Long and Moody defined a machinery to construct representations of braid groups. From a different context, Katz defined a very useful machinery to construct local systems on the complex plane minus n-points which is called Katz ' s middle convolution. In this talk, it will be

explained that these machineries given by Long-Moody and Katz are naturally unified through hypergeometric integrals. This unified machinery will propose an efficient method to obtain irreducible representations of a mixed braid group $B_{1,n}$, and also local systems on various topological spaces, for example, B_n-bundles associated with simple Weierstrass polynomials in the sense of Hansen, complements of hyperplane arrangements of fiber-type, link complements in the solid torus, and so on.

Leonie Mühlherr (Universität Bielefeld)
Title: Recent developments in graphic hyperplane arrangement theory
Abstract: Graphic hyperplane arrangements are an interesting example of arrangements, since they are the subarrangements of the well-studied braid arrangement and have a strong connection to graph theory. This makes it possible to use graph theoretical tools to study them and specifically their module of logarithmic derivations. This talk will survey some concepts and definitions of arrangement classes and their connection to graph theoretical concepts such as separator theory. Some parts of the talk are based on joint work with Takuro Abe, Lukas Kühne and Paul Mücksch.

Shota Maehara (Kyushu University)
Title: Explicit description of a basis for derivation modules of some multiarrangements of type B_2
Abstract: Let us consider multiarrangements in a 2-dimensional vector space over a field of characteristic zero. For 2-dimensional multiarrangements, the exponenets of them are very important to consider the freeness of 3-dimensional simple arrangements. We have some theorems by A. Wakamiko, and by M. Feigin, Z. Wang, and M. Yoshinaga for a multiarrangement whose underlying arrangement is the Coxeter arrangement of type A_2, but there are still many questions left about a basis for the derivation modules and the exponents of multiarrangements in general. In this talk, for multiplicities satisfying some conditions, we show an explicit description of a basis for the derivation module of the Coxeter multiarrangement of type B_2, which is the most simple multiarrangement whose exponents are non-trivial. This is a joint work with Yasuhide Numata.

Alexandru Suciu (Northeastern University)
Title: Hyperplane arrangements with trivial algebraic monodromy
Abstract: To each multi-arrangement $(\mathcal{A}, m)$, there is an associated Milnor fibration of the complement $M = M(\mathcal{A})$. Although the Betti numbers of the Milnor fiber $F = F(\mathcal{A}, m)$ can be expressed in terms of the jump loci for rank 1 local systems on M, explicit formulas are still lacking in full generality, even

for $b_1(F)$. In this talk, I will take a different tack: I will consider the "generic" case (in which $b_1(F)$ is as small as possible), and look deeper into the algebraic topology of such Milnor fibrations. In particular, I will describe ways to extract information on the cohomology jump loci and the lower central series quotients of the fundamental group of F, even when the abelianization of this groups is not torsion-free.

Max Wakefield (The United States Naval Academy)
Title: Chain Tutte polynomials
Abstract: In this talk we will discuss a generalization of the classical Tutte polynomial for matroids. The aim of this study is to define a spectrum of generalized Tutte polynomials to fill the gap between the Tutte polynomial and Derksen's $\mathcal{G}$-invariant. These polynomials are built by taking repeated convolution products of universal Tutte characters studied by Dupont, Fink, and Moci and using the framework of Ardila and Sanchez for studying valuative invariants. We develop foundational aspects of these polynomials by showing they are valuative on generalized permutahedra and present a generalized deletion/contraction formula. We apply these results on chain Tutte polynomials to obtain new formulas for the Möbius polynomial, the opposite characteristic polynomial, a generalized Möbius polynomial, Ford's expected codimension of a matroid variety, and Derksen's $\mathcal{G}$-invariant.

Tan Nhat Tran (Leibniz Universität Hannover)
Title: Vines and MAT-labeled graphs
Abstract: The present talk discusses a connection between two concepts arising from different fields of mathematics. The first concept, called vine, is a graphical model for dependent random variables. This first appeared in a work of Joe (1994), and the formal definition was given later by Cooke (1997). Vines have nowadays become an active research area whose applications can be found in probability theory and uncertainty analysis. The second concept, called MAT-freeness, is a combinatorial property in the theory of freeness of the logarithmic derivation module of hyperplane arrangements. This was first studied by Abe-Barakat-Cuntz-Hoge-Terao (2016), and soon afterwards developed systematically by Cuntz-Muecksch (2020).

In the particular case of graphic arrangements, Tsujie and the speaker recently proved that the MAT-freeness is completely characterized by the existence of certain edge-labeled graphs, called MAT-labeled graphs. In this talk we show that, interestingly, there exists an explicit equivalence between the categories of m-saturated vines and MAT-labeled graphs. In particular, we obtain an equivalence between the categories of regular vines and MAT-labeled complete graphs.

This is joint work with H.M. Tran (Singapore) and S. Tsujie (Hokkaido).

Roberto Pagaria (Bologna University)
Title: Cohomology rings of abelian arrangements
Abstract: We briefly recall the definition of hyperplane arrangements, toric arrangements and their generalization called abelian arrangements. The cohomology ring of the complement is known from a result by Orlik and Solomon (1980) in the hyperplane case. The toric case is due to De Concini and Procesi (2005) and to Callegaro, D ' Adderio, Delucchi, Migliorini, and I (2020). In this talk, we present a new and unified description of the cohomology ring of all abelian (non-compact) arrangements. This is a work in progress with Evienia Bazzocchi and Maddalena Pismataro.

Graham Denham (Western University)
Title: Kirchhoff polynomials and configuration hypersurfaces
Abstract: A finite graph determines a Kirchhoff polynomial, which is a square-free, homogeneous polynomial in a set of variables indexed by the edges. The Kirchhoff polynomial appears in an integrand in the study of particle interactions in high-energy physics, and this provides some incentive to study the motives and periods arising from the projective hypersurface cut out by such a polynomial.

From the geometric perspective, work of Bloch, Esnault and Kreimer (2006) suggested that the most natural object of study is a polynomial determined by a linear matroid realization, for which the Kirchhoff polynomial is a special case.

I will report some recent progress on the interplay between geometry and the combinatorics of arrangements and matroids for this family of objects.

Nir Gadish (The University of Michigan)
Title: Letter-braiding invariants of words in groups
Abstract: How can we tell if a group element is a k-fold nested commutator? For this purpose we introduce the novel invariant theory of words in groups, called letter-braiding invariants: these are elementarily defined functions on words, inspired by the homotopy theory of loop-spaces and carrying deep geometric content. They give a universal finite-type invariant for arbitrary groups, extending the influential Magnus expansion of free groups that already had countless applications in low dimensional topology. As a consequence we get new combinatorial formulas for braid and link invariants, and a way to linearize automorphisms of general groups that specializes to the Johnson homomorphism of mapping class groups.

Zixuan Wang (Hokkaido University)
Title: Integral expressions for multiarrangements
Abstract: This talk is based on the joint work with M. Feigin (University of

Glasgow) and M. Yoshinaga (Osaka University). Terao constructed bases for Coxeter multiarrangements with constant multiplicities. Wakamiko constructed the explicit bases for multiarrangements consisting of three lines by using the generalized binomial coefficients. In fact, it is hard to construct an explicit basis for a free multiarrangement. But more explicit bases for multiarrangements are expected to construct. In this talk, I will use integral expressions to construct a basis for Wakamiko ' s work. I will also discuss the applications of integral expressions for higher-dimensional cases including the free reflection multiarrangements with free multiplicities given by Hoge, Mano, Röhrle, and Stump.

Junyan Chu (Kyushu University)
Title: Minimal Free Resolution of Close-to-free Arrangements
Abstract: We study the algebraic structure of a new class of hyperplane arrangements obtained by deleting two hyperplanes from a free arrangement. Our primary focus is on computing the minimal free resolution of the logarithmic derivation module of such a hyperplane arrangement. In particular, combined with the theory of multiarrangement, we show that the minimal free resolution is determined solely combinatorially for three-dimensional central arrangements. Our result highlights the relationship between algebraic and combinatorial properties for close-to-free arrangements.

Gerhard Röhrle (Ruhr-Universität Bochum)
Title: Invariants and semi-invariants in the cohomology of the complement of a reflection arrangement
Abstract: Suppose V is a finite dimensional, complex vector space, $\mathcal{A}$ is a finite set of codimension one subspaces of V, and G is a finite subgroup of the general linear group $GL(V)$ that permutes the hyperplanes in $\mathcal{A}$. In this talk I we will study invariants and semi-invariants in the graded QG-module $H^*(M(\mathcal{A}))$, where $M(\mathcal{A})$ denotes the complement in V of the hyperplanes in A and $H*$ denotes rational singular cohomology, in the case when G is generated by reflections in V and $\mathcal{A}$ is the set of reflecting hyperplanes determined by G, or a closely related arrangement. The main result consists of the construction of an explicit, natural (from the point of view of Coxeter groups) basis of the space of invariants, $H^*(M(\mathcal{A}))^G$. In addition to leading to a proof of the description of the space of invariants conjectured by Felder and Veselov for Coxeter groups that does not rely on computer calculations, this construction provides an extension of the description of the space of invariants proposed by Felder and Veselov to arbitrary finite unitary reflection groups.

This talk is based on join work with Matt Douglass and Götz Pfeiffer.

Emanuele Delucchi (Scuola Universitaria Professionale Della Svizzera Italiana)
Title: A new arrangement from metric spaces
Abstract: In 2010, Vershik proposed a new combinatorial invariant of metric spaces given by a class of polytopes that arise in the theory of optimal transport and are called "Wasserstein polytopes" or "Kantorovich-Rubinstein polytopes" in the literature. Recently such polytopes have been shown to play an important role in a host of different contexts – however, little is known to date about their structure. In particular, Vershik asked about a description of the polytope's features in terms of the metric space, and about the stratification of the space of all metrics according to the combinatorial type of such polytopes.

After stating the definitions and some examples in the case of finite metric spaces, in this talk I will present a class of arrangements that describe the stratification sought by Vershik. To the best of our knowledge, such arrangements have not been studied before: I will mention some open problems about them that are geared towards advancing our understanding of Vershik's questions.

The talk is based on joint work with Lukas Kühne, Leonie Mühlherr and Linard Hössly.

Christin Bibby (Louisiana State University)
Title: Supersolvable posets and fiber-type abelian arrangements
Abstract: We present a combinatorial analysis of fiber bundles of generalized configuration spaces on connected abelian Lie groups. These bundles are akin to those of Fadell–Neuwirth for configuration spaces, and their existence is detected by a combinatorial property of an associated finite partially ordered set. This is consistent with Terao's fibration theorem connecting bundles of hyperplane arrangements to Stanley's lattice supersolvability. We obtain a combinatorially determined class of $K(\pi, 1)$ toric and elliptic arrangements. Under a stronger combinatorial condition, we prove a factorization of the Poincaré polynomial when the Lie group is noncompact. In the case of toric arrangements, this provides an analogue of Falk–Randell's formula relating the Poincaré polynomial to the lower central series of the fundamental group. This is joint work with Emanuele Delucchi.

Sven Wiesner (Ruhr-Universität Bochum)
Title: Free multiderivations of connected subgraph arrangements
Abstract: Recently Cuntz and Kühne introduced a hyperplane arrangement $\mathcal{A}(G)$ associated with a given connected graph G and they classified all graphs G such that $\mathcal{A}(G)$ is a free arrangement. I will present joint work with Gerhard Röhrle and Paul Mücksch in which we generalize this result to the multiarrangement case. We give a complete list of graphs which possess at least one non-trivial multiplicity m such that the corresponding multiarrangement $(\mathcal{A}(G), m)$ is free.

Georges Neaime (Universität Bielefeld)
Title: Reflection Groups and Artin Groups: Classical and Elliptic Theories
Abstract: We will briefly describe the theory of elliptic Weyl groups and elliptic Artin groups, as established by Kyoji Saito in the '80s. We will compare this description with the classical theory of reflection groups and their Artin groups. We will also mention some recent developments in the elliptic theory, as well as some applications to hereditary categories.

Lukas Kühne (Universität Bielefeld)
Title: Computing the moduli space of a matroid and applications to arrangements
Abstract: A matroid is a fundamental and widely studied object in combinatorics. Following a brief introduction to matroids, I will showcase parts of a new OSCAR module for matroids using several examples. My emphasis will be on the computation of the realization space of a matroid, which is the space of all hyperplane arrangements that have the given matroid as their intersection lattice.

Later, I will discuss its applications in the realm of hyperplane arrangements. First, I will outline a connection between matroid realization spaces, operators that act on line arrangements, and elliptic modular surfaces. Time permitting, I will report on an ongoing project to construct line arrangements with only triple intersection points.